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Kazimierz OPOKA[1]

PREDICTING ECONOMIC INDICES OF VEHICLE INSURANCE USING THE "GREY-SYSTEM THEORY"

Summary. This paper contains a prognosis of vehicle insurance economic indices using the Grey System Theory. It has been prepared based on data provided by a certain insurance company. The following economic factors were analysed: number of insured vehicles, premiums income, amount of damage cases covered, and value of paid compensations. The results of this study indicate a reduction in all analysed indices over twelve months.
Keywords: transport, grey-system theory, insurance

1. INTRODUCTION

Continuous increase in vehicle use all over the world, including Poland, is accordingly followed by rapid growth in the number of road accidents [1-3]. These result in injuries among not only car users, but also other public road users, for example, pedestrians, cyclists, etc. Despite a thorough and constantly improving prevention policy, road accidents cannot be eliminated, thus emphasis should be put on mitigating their consequences, among others, by enforcing better legislation that is critical for higher safety on public roads [4-5]. The most important regulations that affect safety on public roads, and in particular, protect the casualties of road accidents include the proper formulation of rules regarding civil liability for car accidents and effective insurance.

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[1] State Higher Vocational School in Nowy Sącz, 1a Zamenhofa Street, 33-300 Nowy Sącz, Poland. Email: slawkow1@op.pl

The concept of vehicle insurance refers to all types of insurance regarding motor vehicles. These include [6]:

- mandatory civil liability insurance of motor vehicle users, governed by the Act of 22 May 2003 on compulsory insurance, the Insurance Guarantee Fund and Polish Motor Insurers' Bureau,
- other voluntary vehicle insurance including vehicle damage consequences and theft insurance (referred to as the "autocasco insurance" in Poland), all-accidents insurance of driver and passengers.

Currently, 56 notified insurance companies operate in Poland.

Since the major economic indices of the insurance industry are the income on the sale of insurance policies and the amount of paid compensations, this analysis shall predict the following indices:

- number of insured vehicles,
- premium income,
- number of damage cases covered,
- value of paid compensations.

The input parameters of the prognosis were the data provided by a certain insurance company operating in Poland. These data concerned the period from 2018 to 2019 and the prognosis concerned the following 12 months.

2. DISTRICT INFRASTRUCTURE AND VEHICLE ACCIDENTS RATES

Road accidents are categorised as random events. However, it must be pointed out that their quantity and severity are certainly determined by terrain conditions, including road infrastructure, as well as the number of vehicles in traffic.

2.1. Geographical situation, classification and number of communication routes

Nowy Sącz district is situated in the southeastern part of the Małopolskie Voivodeship. On the south, it borders with Slovakia (state border), on the east with Gorlice district, on the north with Tarnów and Brzesko district and on the west with Limanowa and Nowy Targ district. The total area of the district is 1550.11 km^2. Mountains and uplands, as well as the valleys of the Dunajec River and its main tributaries: Poprad and Kamienica, cover most of the area. These rivers separate the major mountain ranges of the Nowy Sącz region: Beskid Sądecki, Low Beskids and Island Beskids surrounding the Sądecka Valley, the largest settlement area of the region. The major towns of the district are Nowy Sącz (capital), Stary Sącz and Grybów. Other known towns and spa communes include Krynica, Muszyna and Piwniczna. There are also smaller communes and villages, including Podegrodzie, Łącko, Chełmiec, Nawojowa, Korzenna, Kamionka Wielka, Rytro and Łososina Dolna.

Tab. 1 presents the classification, number and lengths of major roads approved for wheeled traffic in the district.

Tab. 1

Classification, number and lengths of roads

Road category	Number of roads	Road length [km]
State	3	96.9
Regional	4	118.2
District	80	5.0

2.2. Summary of registered vehicles

Tab. 2 presents a summary of vehicles registered in the Nowy Sącz district as at 31 December 2018 and 30 November 2019.

Tab. 2

Summary of vehicles registered from 2009 to 2018 [7]

Years	Motor vehicles		Vehicle categories					
			Cars		Lorries		Motorcycles	
	Total	2009=100 %	Total	2009=100 %	Total	2009=100 %	Total	2009=100 %
2009	22,024,697	100.0	16,494,650	100.0	2,595,485	100.0	974,906	100.0
2010	23,037,149	104.6	17,239,800	104.5	2,767,035	106.6	1,013,014	103.9
2011	24,189,370	109.8	18,125,490	109.9	2,892,064	111.4	1,069,195	109.7
2012	24,875,717	112.9	18,744,412	113.6	2,920,779	112.5	1,107,260	113.6
2013	25,683,575	116.6	19,389,446	117.5	2,962,064	114.1	1,153,169	118.3
2014	26,472,274	120.2	20,003,863	121.3	3,037,427	117.0	1,189,527	122.0
2015	27,409,106	124.4	20,723,423	125.6	3,098,376	119.4	1,272,333	130.5
2016	28,601,037	129.9	21,675,388	131.4	3,179,655	122.5	1,355,625	139.1
2017	29,149,178	132.3	22,109,572	134.0	3,212,690	123.8	1,398,609	143.5
2018	29,656,238	134.6	22,514,074	136.5	3,249,961	125.2	1,428,299	146.5

The table above indicates that the number of vehicles registered in Poland grows dynamically. This increase is apparent on the roads and streets.

Due to the tourist value of the Nowy Sącz region, as well as the multitude of transport companies operating in the area, road traffic is very intense. Considering the hilly road configuration, with many sharp bends and slopes, particular caution is needed when driving across the region, irrespective of the season of the year or time of the day, with a special concern for weather conditions.

2.3. Road accidents and collisions as at 31 December 2018

An accident is an unfortunate random event in which participants die, suffer from health impairment or material damage. Road incidents are divided into road collisions and road accidents.

Classification of road incident in one of the mentioned categories depends on the type of injury of participants and is specified in the Polish Code of Offences (Art. 156 Sections 1, 2, 3, Art. 157 Sections 1, 2, Art. 177 Section 1).

These regulations provide the following definitions:

- collision – event in which no injury occurs or that results in injuries diagnosed by an authorised doctor, lasting no longer than 7 days,
- accident – event in which participants die or suffer health impairment diagnosed by an authorised doctor, lasting longer than 7 days.

Available statistics published by the National Road Safety Board gave the following main reasons for occurrence of road incidents in 2019:

- failure to yield the right of way,
- inappropriate speed in particular traffic conditions,
- failure to yield the right of way at a pedestrian crossing,
- failure to keep a safe distance between vehicles,
- improper passing.

The most dangerous roads in the Nowy Sącz district were:

- national road No. 75 (route from Witowice to Krynica),
- national road No. 28 (route from Wysokie to Ropska Góra),
- regional road No. 971 (route from Krynica to Piwniczna),
- national road No. 87 (route from Nowy Sącz to Piwniczna),
- regional road No. 969 (route from Zabrzeż to Nowy Sącz).

These routes are presented in Fig. 1. Fig. 2 presents the trend in road accidents occurrence, while the number of fatal casualties from 2009 to 2018 is shown in Fig. 3.

Comparing the data contained in Tab. 2 representing a significant increase in the number of registered vehicles with the road accident rates illustrated in the diagrams, it can be concluded that there is an apparent drop in the number of accidents, including fatal accidents.

2.3. Compulsory vehicle insurance

The vehicle insurance sector has been developing in Poland after World War II. An indispensable part of the state reconstruction was the rapid growth of mobility. This process and related occurrence of accidents due to the ever-increasing number of vehicles were followed by a significant increase in the rate of material (vehicles) and personal damage (bodily injury, health impairment, death) among public roads users. Compensation usually was beyond the capabilities of the owners and drivers of motor vehicles, thus casualties incurred significant and irreversible material losses.

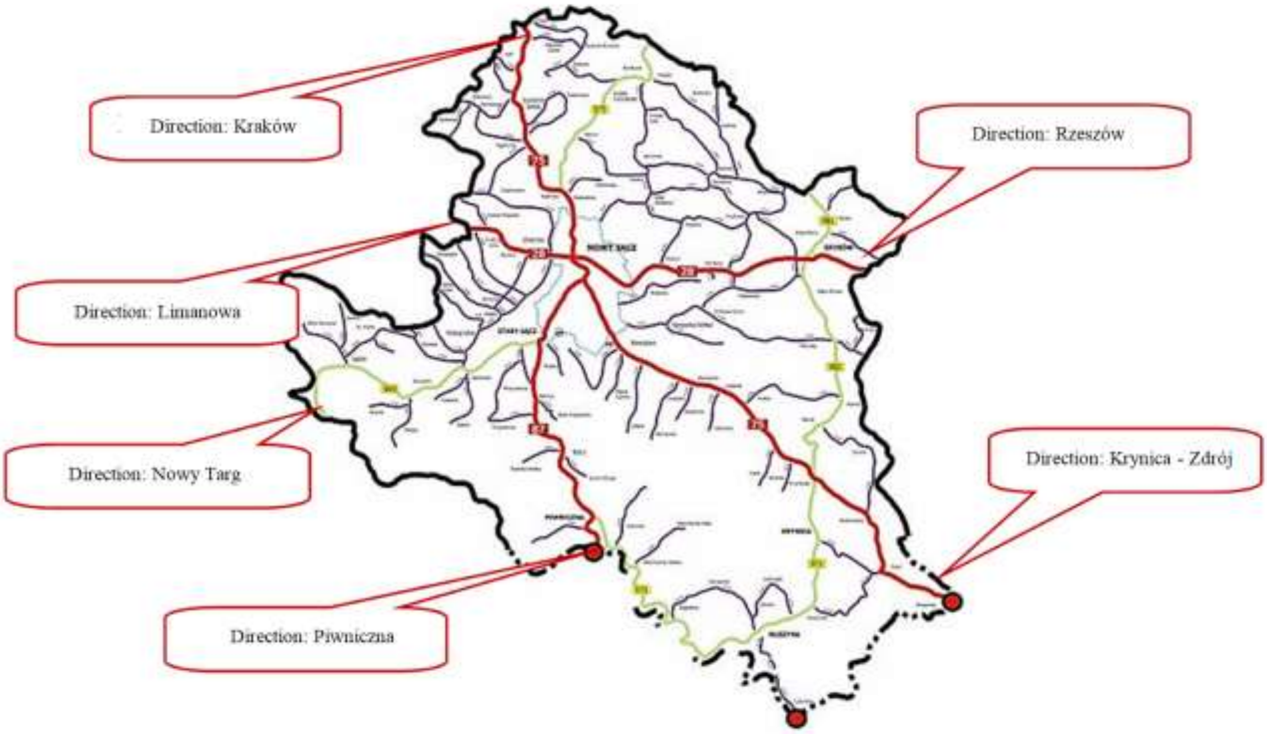

Fig. 1. Nowy Sącz district – most dangerous routes

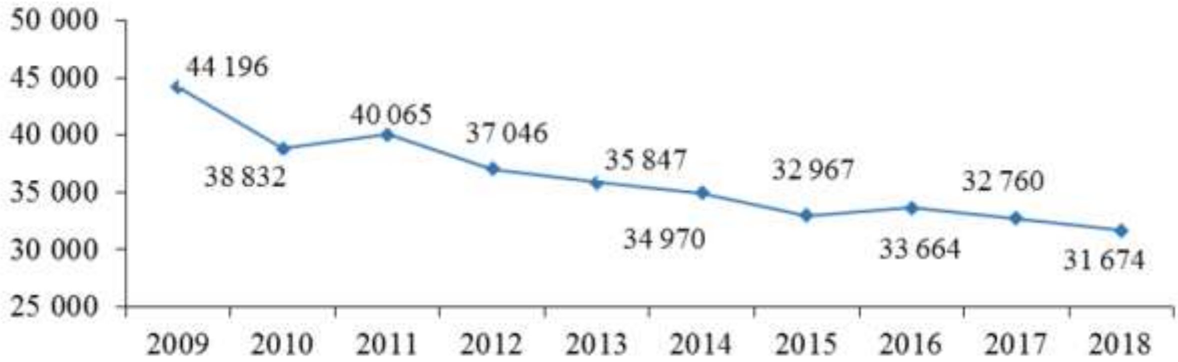

Fig. 2. Road accidents occurrence trend from 2009 to 2018 year [7]

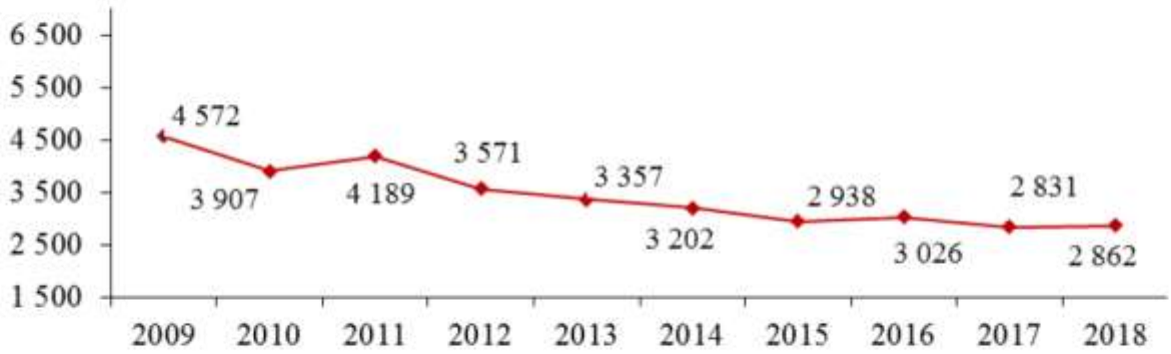

Fig. 3. Road accidents fatal casualties trend from 2009 to 2018 year [7]

To secure accident casualties against damage, the Act of 2 December 1958 on property and personal insurance (Journal of Laws No. 72, item 357, Art. 5, as amended) introduced two types of compulsory insurance:

- civil liability insurance of owners and drivers of motor vehicles against damage caused in traffic,
- insurance against all accidents to passengers or other casualties in motor vehicle traffic.

Over the years, compulsory vehicle insurance rules have been updated and revised. Currently, it is governed by the Act of 22 May 2003 on compulsory insurance, the Insurance Guarantee Fund and Polish Motor Insurers' Bureau. According to Art. 23 thereof, the owner of a motor vehicle is obliged to sign a compulsory liability insurance agreement covering damage caused in relation to participation in traffic.

Damage occurs as a consequence of random events. The general legal definition of “damage” says that it consists of a breach of legally protected property and interest, which can only be repaired if other liability requirements are fulfilled. Additionally, damage entails any material or non-material losses.

In the analysed period, the insurance company participating in the study registered 15,458 damage cases.

3. GREY SYSTEM THEORY

The Grey System Theory was developed in the early 1980s. The concept refers to an operational condition of an item, in which a certain amount (part) of data describing a given parameter is known (that is, is clearly defined), while the other part is unknown. The “Grey System Theory” method uses the known part of information to determine (predict) the unknown part. The character of the system is controlled in advance to shape its future development. The Grey System Theory is used in forecasting events in economics and agriculture. It was originally used for machine health monitoring and troubleshooting. This prediction theory works mainly with past data to create a mathematical model simulating time cycle data. If the required measurement accuracy cannot be obtained, it is compensated and rectified by identifying the “remaining data” until proper prediction results are reached.

The Grey System Theory has been applied in many fields of science and life. Wang et al. (2020), used this method to predict the municipal demand for heat production [8]. The authors of [9] predicted highway traffic intensity, while authors of [10] dealt with time series prediction applied to road traffic safety in Germany. The Grey System Theory has been applied in the health care sector [11], in surface engineering to predict pitting fatigue [12], and also in logistics, to support freight delivery related decisions [13] or in prediction of diagnostic symptom values [14].

The model of grey system is described with the following differential equation:

$$\frac{dX^{1}(t)}{dt} + aX^{(1)}(t) = u \qquad (1)$$

Parameters “a” and “u” can be calculated by determining the $X^1(k)$ model, previously denoted as the sum of input values of $X^0(t)$ models for i=l,2,...,k.

$$X^{(n)}(k) = \sum_{i=1}^{k} X^{(n-1)}(i) \tag{2}$$

The Grey System Model derives from the obtained data. Then, the answer or predictive equation has the following form:

$$X^{(1)}(t+1) = \left(X^{(1)}(0) - \frac{u}{a} \right) \cdot e^{-at} + \frac{u}{a} \tag{3}$$

Parameters “a” and “u” can be calculated using the following equation:

$$\hat{a} = (a,u)^T$$

according to the least squares method:

$$\hat{a} = (a,u)^T = (B^T B)^{-1} B^T Y_n \tag{4}$$

where B is the following data matrix:

$$B = \begin{bmatrix} -\frac{1}{2}(X^{(1)}(1) + X^{(1)}(2)) & 1 \\ -\frac{1}{2}(X^{(1)}(2) + X^{(1)}(3)) & 1 \\ \dots\dots\dots\dots\dots\dots & \\ \dots\dots\dots\dots\dots\dots & \\ \dots\dots\dots\dots\dots\dots & \\ -\frac{1}{2}(X^{(1)}(n-1) + X^{(1)}(n)) & 1 \end{bmatrix} \tag{5}$$

$$Y_n = [X^{(0)}(2) \quad X^{(0)}(3) \dots\dots X^{(0)}(n)]^T \tag{6}$$

If the required model accuracy cannot be reached, it is compensated and rectified by identifying the “remaining data” of the model.

The following data strings will be determined using Equation 3:

$$\hat{X}^{(1)}(t) = (\hat{X}^{(1)}(1), \hat{X}^{(1)}(2), \dots, \hat{X}^{(1)}(n))^T$$

Predicted data are calculated in the following way:

$$\hat{X}^{(0)}(t+1) = \hat{X}^{(1)}(t+1) - \hat{X}^{(1)}(t)$$

$$\hat{X}^{(0)}(t+1) = (\hat{X}^{(0)}(0) - \frac{u}{a}) \cdot e^{-a(t-1)} \cdot (e^{-a} - 1) \tag{7}$$

The rest of the cycle is calculated using the following equation:

$$\{\varepsilon_0(k)\} = \{\hat{X}^{(0)}(k) - \hat{X}^{(0)}(k)\} \tag{8}$$
$$k=1,2,...,n$$

Having calculated the rest of the cycle $\{\varepsilon^{(0)}(k)\}$, other predicted values can be obtained using Equation 3. Next, the values of the remaining part of the prognosis should be added to the results of prediction $X^{(0)}(k)$. This process can be repeated as many times as required until model accuracy is reached.

The accuracy of the described method is determined using the following equation, referred to as "remaining data":

$$q(k) = x^{(0)}(k) - x^{(0)}(k) \tag{9}$$

where q(k) is the rest of k data.

The average value of real data X0(k) is denoted in the following form:

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k),\ k=1,2,...,n \tag{10}$$

The average value of the "remaining data" q(k) is denoted in the following form:

$$\bar{q} = \frac{1}{n}\sum_{k=1}^{n'} q(k),\quad n' < n,\quad k=1,2,...,n \tag{11}$$

The variance of real (measured) values S:

$$S_1^2 = \frac{1}{n}\sum_{k=1}^{n}(x^{(0)}(k) - \bar{x})^2 \tag{12}$$

The variance of the "remaining data" S:

$$S_2^2 = \frac{1}{n}\sum_{k=1}^{n}(q(k) - \bar{q})^2 \tag{13}$$

The quotient of these two values:

$$C = \frac{S_2}{S_1} \tag{14}$$

is referred to as the discrepancy quotient.

Calculated values C (Tab. 3) correspond to the lowest error probability values P:

$$P = p\{|q(k) - \bar{q}| < 0{,}6745 S_1\} \tag{15}$$

which is equivalent to classification of the prognosis in a proper accuracy group.

Tab. 3

Method accuracy prediction

Prognosis accuracy	*P*	*C*
good	>0.95	<0.35
satisfactory	0.8-0.9	0.35-0.5
unsatisfactory	0.7-0.8	0.5-0.65
poor	<0.7	>0.65

In the analysed sample predictions based on the "Grey System Theory", predicted and actual data (obtained in measurements) are quite consistent. The Grey System Theory can be used in technical facilities condition inspections as one of the monitoring system elements.

4. PREDICTING ECONOMIC INDICES IN VEHICLE INSURANCE

4.1. Presentation of processed data

As earlier mentioned in the introduction section, the prognosis was prepared using statistical data recorded by a certain insurance company operating in Poland, at the end of each four-month settlement period in 2018 and 2019. Fig. 4 presents the insurance economic indices included in the described prognosis.

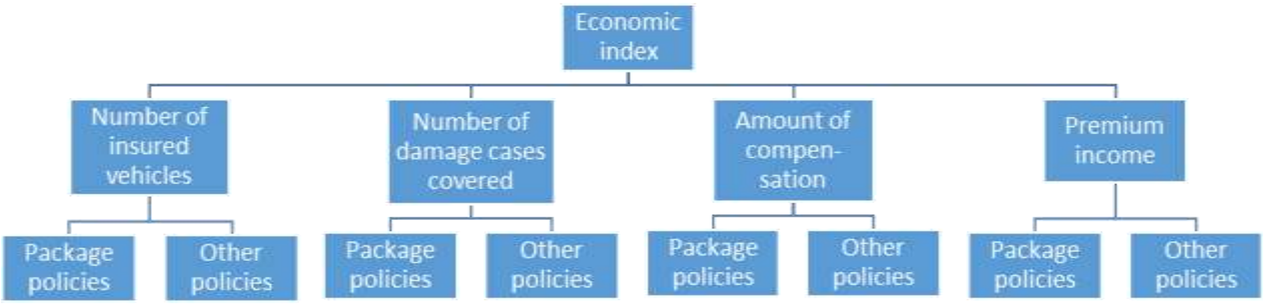

Fig. 4. Scheme of predicted insurance economic indices

- Package policies - provide comprehensive coverage of the vehicle and its owner and include compulsory liability insurance, voluntary all-accident insurance with Assistance insurance (technical support and medical aid in Poland), post-accident coverage for driver and passengers. Offered to owners of cars, vans and goods vehicles with gross vehicle weight up to 2 [t] in the analysed insurance company.
- Other policies - including all-accident insurance.
- Number of insured vehicles - number of insurance agreements registered in the analysed period.
- Premium income - amount earned on the sale of insurance policies in the analysed period.
- Amount of compensation - amount of benefits paid in the analysed period.

- Number of damage cases covered - number of damage cases compensated in the analysed period.

Tab. 4 contains a summary of the input data needed by insurance companies for qualitative and quantitative prediction of road accidents.

Tab. 4

Summary of input data

No.	Period of analysis	Number of insured vehicles	Premium income [PLN]	Number of damage cases covered	Amount of compensation [PLN]
Vehicles with package insurance policies					
1	1st quarter of 2018	1,433	1.766,350	188	760,864
2	2nd quarter of 2018	1,729	2,104,945	170	678,343
3	3rd quarter of 2018	1,369	1,643,956	255	787,276
4	4th quarter of 2018	1.106	1,289,829	289	1,175,124
5	1st quarter of 2019	1,044	1,244,893	331	985,636
6	2nd quarter of 2019	1,019	1,292,606	260	878,857
7	3rd quarter of 2019	922	1,162,542	269	1,086,480
8	4th quarter of 2019	651	800,573	199	599,693
Vehicles with other insurance policies					
1	1st quarter of 2018	1,130	1,047,601	274	1,225,717
2	2nd quarter of 2018	1,293	1,336,660	245	1,150,498
3	3rd quarter of 2018	1,095	1,117,935	199	915,445
4	4th quarter of 2018	1,144	1,049,565	178	813,200
5	1st quarter of 2019	1,362	1,375,988	214	1,216,293
6	2nd quarter of 2019	1,417	1,491,511	262	873,208
7	3rd quarter of 2019	1,356	1,432,006	289	1,441,465
8	4th quarter of 2019	9,095	10,235,056	267	1,134,300

4.2. Prognosis of economic indices of vehicle insurance

Tab. 5 contains a summary of input data divided into separate economic indices. Data is presented in the descending order.

Tab. 5

Input data in descending order

	I	II	III	IV	V	VI	VII	VIII
Number of damage cases covered – package policies								
	331	289	269	260	255	199	188	170
Number of damage cases covered – other policies								
	289	274	267	262	245	214	199	178
Amount of compensation – package policies								
	117,5124	108,6480	985,636	878,857	787,276	760,864	67,8343	599,693

Amount of compensation – other policies								
	144,1465	1,225,717	1,216,293	1,150,498	1,134,300	915,445	873,208	813,200
Premium income – package policies								
	2,104,945	1,766,350	1,643,956	129,606	1,289,829	1,244,893	1,162,542	800,573
Premium income – other policies								
	1,491,511	1,434,937	1,432,006	1,375,988	1,336,660	1,117,935	1,049,565	1,047,601
Number of insured vehicles – package policies								
	1,729	1,433	1,369	1,106	1,044	1,019	922	651
Number of insured vehicles – other policies								
	1,417	1,362	1,359	1,324	1,293	1,144	1,130	1,095

Using the formulas presented in the previous section, prognosis equations for individual economic indices was derived.

Number of damage cases covered – package policies

$$\hat{X}_0(t+1) = 297{,}8325524 \cdot e^{-0{,}087007235(t-1)}$$

Number of damage cases covered – other policies

$$\hat{X}_0(t+1) = 286{,}1649775 \cdot e^{-0{,}070094172(t-1)}$$

Amount of compensation – package policies

$$\hat{X}_0(t+1) = 1079484{,}617 \cdot e^{-0{,}095789794(t-1)}$$

Amount of compensation – other policies

$$\hat{X}_0(t+1) = 1285959{,}966 \cdot e^{-0{,}071946324(t-1)}$$

Premium income – package policies

$$\hat{X}_0(t+1) = 1764726{,}951 \cdot e^{-0{,}10569395(t-1)}$$

Premium income – other policies

$$\hat{X}_0(t+1) = 1498074{,}86 \cdot e^{-0{,}061143908(t-1)}$$

Number of vehicles insured – package policies

$$\hat{X}_0(t+1) = 1459{,}535617 \cdot e^{-0{,}108958939(t-1)}$$

Number of vehicles insured – other policies

$$\hat{X}_0(t+1) = 1401{,}492193 \cdot e^{-0{,}040906471(t-1)}$$

Tab. 6 contains a summary of input data and predicted values calculated using the equations presented above. A – stands for input value, B – predicted value. Figs. 5 to 8 present the distribution of analysed economic indices of the insurance business.

Tab. 6

Summary of input data and predicted values

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Number of damage cases covered – package policies												
A	331	289	269	260	255	199	188	170	-	-	-	-
B	331	298	273	250	229	210	192	176	162	148	136	125
Number of damage cases covered – other policies												
A	289	274	267	262	245	214	199	178	-	-	-	-
B	289	286	267	249	232	216	201	188	175	163	152	142

Amount of compensation – package policies x 1000												
A	1,175	1,086	985	878	787	760	678	599	-	-	-	-
B	1,175	1,079	980	891	809	735	668	607	552	501	455	414
Amount of compensation – other policies x 1000												
A	1,441	1,225	1,216	1,150	1,134	915	873	813	-	-	-	-
B	1,441	1,285	1,196	1,113	1,036	964	897	835	777	723	672	626
Premium income – package policies x 100												
A	21,049	17,663	16,439	12,926	12,898	12,448	11,625	8,005	-	-	-	-
B	21,049	17,647	15,877	14,284	12,851	11,562	10,403	9,359	8,420	7,576	6,816	6,132
Premium income – other policies x 100												
A	14,915	14,349	14,320	13,759	13,366	11,179	10,495	10,476	-	-	-	-
B	14,915	14,980	14,092	13,256	12,470	11,730	11,034	10,380	9,764	9,185	8,640	8,128
Number of insured vehicles – package policies												
A	1,729	1,433	1,369	1,106	1,044	1,019	922	651	-	-	-	-
B	1,729	1,459	1,309	1,174	1,052	944	846	759	681	610	547	491
Number of insured vehicles – other policies												
A	1,417	1,362	1,359	1,324	1,293	1,144	1,130	1,095	-	-	-	-
B	1,417	1,401	1,345	1,291	1,239	1,190	1,142	1,096	1,052	1,010	969	931

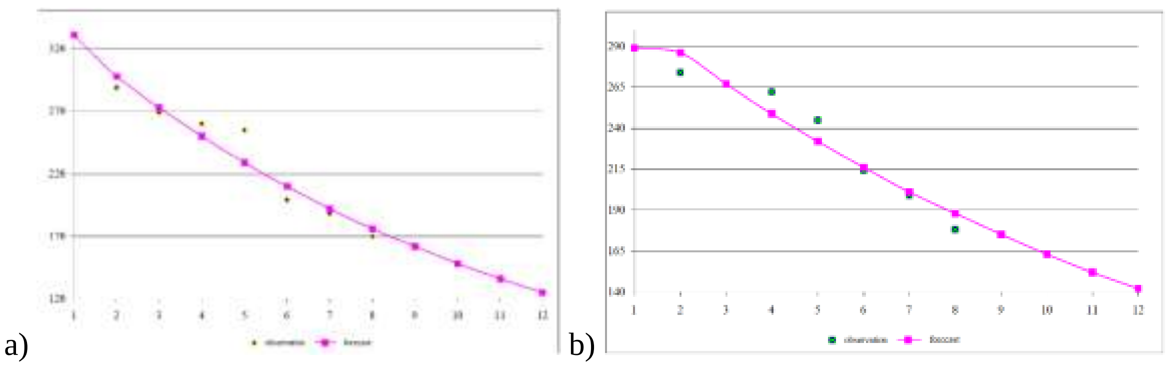

Fig. 5. Number of damage cases covered a) package policies, b) other policies

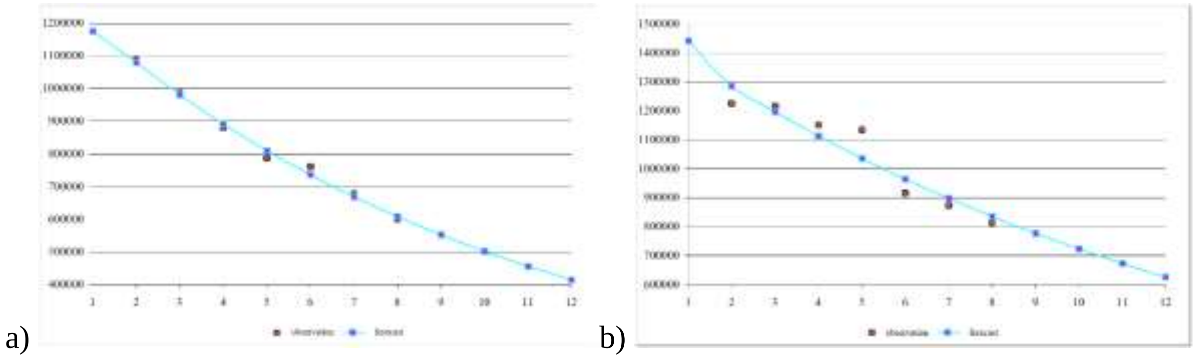

Fig. 6. Amount of paid compensation a) package policies, b) other policies

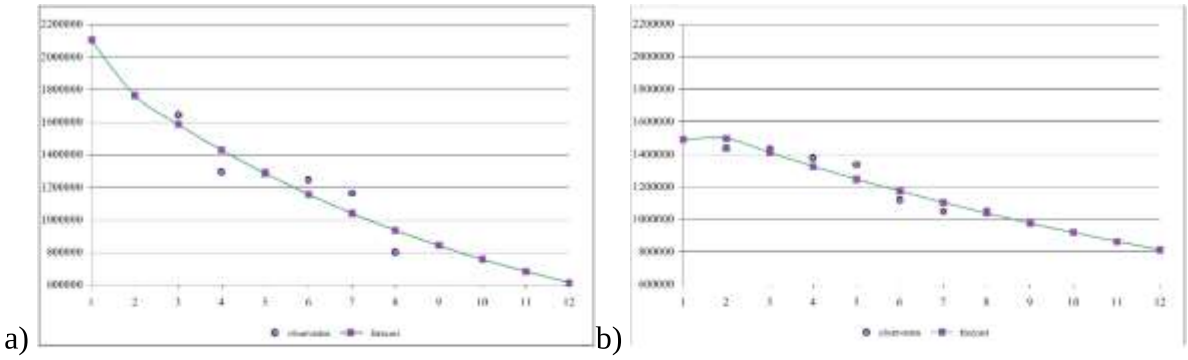

Fig. 7. Premium income amount a) package policies, b) other policies

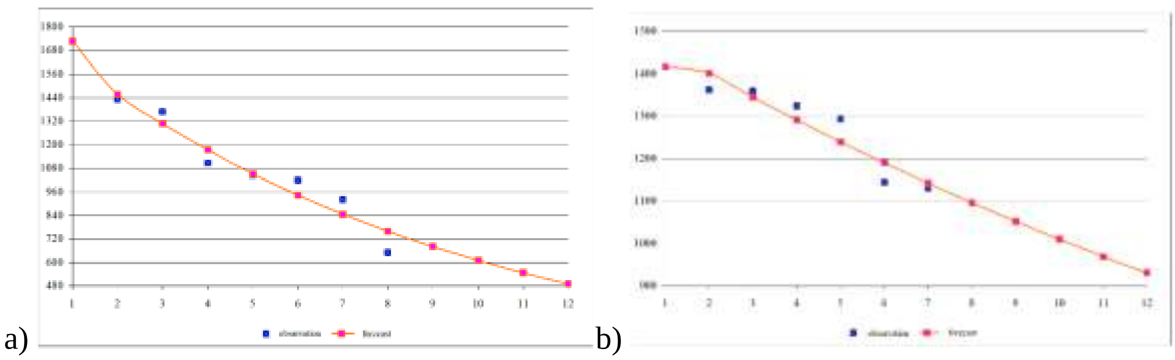

Fig. 8. Number of insured vehicles a) package policies, b) other policies

The accuracy of the method has been determined using Equations 10 to 13. Probability P for each $|q(k)-\bar{q}|$ exceeds 0.95 in all analysed indices. This posits that the proposed method is correct.

5. CONCLUSIONS

The results presented in Section 4 indicate that the prognosis suggested a probable decrease in the value and quantity of each analysed economic index of vehicle insurance.

The decreasing trend is reasonably related to the number of insurance policies and the amount of paid compensation. The number of newly registered cars for which liability insurance is legally required is constantly increasing. Road statistics with reference to the number of cars cited in 2.3 above show a decreasing trend. The value of paid compensation is largely affected by the post-accident repair cost estimate. The number of used cars imported to Poland is still high. Although their age varies, most cars are several years old. Costs of accident repairs in these vehicles quite often exceed their market value, which is reflected in lower amounts of paid compensation.

Currently, monitoring of the analysed indices is about to start so as to verify if the results obtained using the Grey System Theory method correspond to the actual economic situation in 2020.

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