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FORMULATION OF MATHEMATICAL AND PHYSICAL MODEL OF PNEUMATIC FLEXIBLE SHAFT COUPLINGS

Summary. On our department we deal with the research and development of pneumatic flexible shaft couplings. The aim of this paper is to present the way to determine the static load characteristics of pneumatic coupling. By this method, the pneumatic flexible shaft coupling is described by a mathematical and physical model which parameters are determined from the results of static load characteristics measurement.

Keywords. pneumatic flexible shaft coupling, pneumatic springs, static properties, volume of pneumatic springs, mathematical and physical model.

TWORZENIE MATEMATYCZNO-FIZYCZNEGO MODELU PNEUMATYCZNYCH SPRZĘGIEŁ ELASTYTYCZNYCH ŁĄCZĄCYCH WAŁY

Streszczenie. W naszej pracowni zajmujemy się badaniami i rozwojem pneumatycznych sprzęgieł elestycznych łączących wały. Celem artykułu jest przedstawienie nowego sposobu określania właściwości statycznych pneumatycznych sprzęgieł elestycznych łączących wały. W podanym sposobie pneumatyczne sprzęgło elastyczne łączące wały jest opisane przez model matematyczno-fizyczny, którego parametry są wyznaczone na podstawie wyników pomiarów eksperymentalnych jego statycznych charakterystyk obciążenia.

Słowa kluczowe. pneumatyczne sprzęgło elastyczne łączące wały, sprężyny pneumatyczne, właściwości statyczne, objętość sprężyn pneumatycznych, model matematyczno-fizyczny.

1. INTRODUCTION

On our department we deal with the research and development of pneumatic flexible shaft couplings [3], [6], [7]. Pneumatic flexible shaft couplings allow the change of their stiffness by the pressure change of gaseous medium in their pneumatic flexible elements. The torsional stiffness of flexible coupling has a high influence on the load torque [8], noise and lifetime of mechanical drive in which it is applied. The static properties of already manufactured pneumatic couplings are determined experimentally by measuring of their static load characteristics by selected pressures of gaseous medium [1]. This experimental method

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gives accurate and reliable results. Its disadvantage is that physical processes in the pneumatic elements by the distortion of the coupling are not taken in the account. It means the necessity of a relatively large number of measurements which be used only for one specific of the coupling's compression space design. The properties of flexible couplings on design are determined by a mathematical and physical model based on isothermal pressure changes in the compression space of pneumatic flexible elements [2]. The disadvantage of this method is that the exact compression space volume and influence of rubber shell of the pneumatic elements are unknown. The advantage is that the static properties of pneumatic coupling can be computed very quickly from the twist angle and static pressure of gaseous medium. The aim of this paper is to present the way to determine the static load characteristics of pneumatic coupling, by combining the two previous methods.

2. INVESTIGATED PNEUMATIC FLEXIBLE SHFAFT COUPLING

Pneumatic flexible shaft coupling type *3 – 1/110 – T – C* manufactured by *FENA* company was used for the verification of the accuracy and description of the new method. This pneumatic flexible coupling (*Fig. 1*) consists of driving hub *(1)* and a driven hub *(2)*, connected by three pneumatic flexible elements *(4)*. The compression spaces of pneumatic elements are interconnected by tubes *(3)*. Because the flexible elements of this coupling are always simultaneously compressed or expanded, this coupling can transmit the torque only in one direction (in the direction of compression). The properties of this coupling were determined experimentally at our department [4]. The measured static load characteristics are presented on (*Fig. 2*).

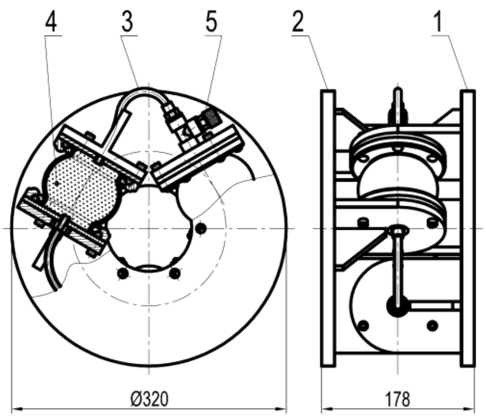

Fig. 1. Pneumatic flexible shaft coupling type 3 – 1/110 – T – C
Rys. 1. Elastyczne sprzęgło pneumatyczne łączące wały typu 3 – 1/110 – T – C

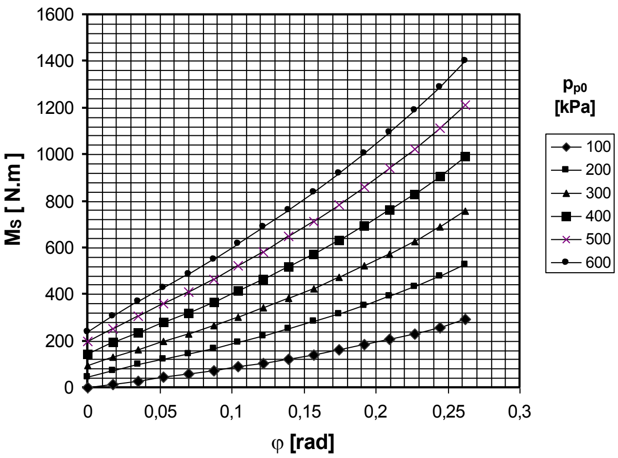

Fig. 2. Static load characteristics
Rys. 2. Charakterystyki statyczne obciążenia

In the present the static characteristics of pneumatic flexible shaft couplings are determined in the same way as for common flexible shaft couplings with rubber flexible elements. So the dependency of static load torque on the static twist angle is measured. For pneumatic flexible shaft couplings it is necessary to perform these measurements by different values of initial overpressure of gaseous medium in the compression volume. The initial overpressure is the overpressure by initial (zero) twist angle. If necessary (determining the static characteristics by different overpressure, static stiffness, non-linearity coefficient of static load characteristics, ...) it is possible to describe the measured static load characteristic by a regression curve (usually third degree polynomial) and then determine the dependency of curve parameters on the initial overpressure (also by suitable regression curves).

However the following physical phenomena are not taken into account:

- By twisting the coupling the volume of gaseous medium in the compression space changes. It is not possible to set the pressure by other twist angle than the initial twist angle.
- If the volume of compression space is modified then the static load characteristics of pneumatic coupling will be also different.
- The accurate description of transient states caused by pressure changes (by filling up or by discharging the compression space with gaseous medium) is not possible.

3. WORK PRINCIPLE OF PNEUMATIC FLEXIBLE SHAFT COUPLINGS

The torque of pneumatic flexible shaft coupling M_S is a sum of pneumatic flexible element rubber shell torque M_G and torque M_V from the overpressure of gaseous medium enclosed in the compression space of coupling (1). The absolute value of the pressure p can be determined as a sum of overpressure of gaseous medium in compression space p_p and atmospheric pressure $p_a = 1.10^5$ Pa (2).

$$M_S = M_G + M_V \qquad (1) \qquad\qquad p = p_a + p_p \qquad (2)$$

Changing the twist angle of pneumatic flexible shaft coupling by $d\varphi$ causes a change of compression space volume by a volume dV. For the work performed by torque M_V from the overpressure of gaseous medium and the work of compressed gaseous medium applies (3). For the volume of compression space applies (4).

$$M_V \cdot d\varphi = -p_p \cdot dV \qquad (3) \qquad\qquad \frac{dV}{d\varphi} = -S_e \cdot r \qquad (4)$$

Where:

S_e [m^2] – effective area of the coupling compression space,
r [m] – distance of the center of effective area S_e from the coupling axis.

Expression $Se.r$ is then the static moment of effective area S_e to the coupling axis.

Then the coupling torque from the overpressure of gaseous medium will be:

$$M_V = p_p \cdot S_e \cdot r \qquad (5)$$

From (5) is clear that the coupling torque from the overpressure depends on the overpressure of gaseous medium and on the static moment of effective area of compression space. We assume that the volume of compression space as well as the effective area of pneumatic elements by given twist does not depend on the overpressure. We also assume that the torque from rubber shell M_G depends only on the twist angle φ.

4. DETERMINING THE PARAMETERS OF MATHEMATICAL AND PHYSICAL MODEL OF PNEUMATIC FLEXIBLE SHAFT COUPLING

For the use of method described in this article it is necessary to measure also the dependency of the overpressure of gaseous medium on the twist angle. (*Fig. 3*).

As it was mentioned the torque of rubber shell of M_G, as well as the static moment of effective area $Se.r$ of compression space depend only on the twist angle φ.

Therefore we need to get the dependency of the coupling static torque M_S on overpressure p_p by constant twist angles φ. From these curves by linear regression it is possible to determine the parameters of formula (6), i.e. the torque of rubber shell M_G (*Fig. 4*) and the static moment of effective area of compression space $Se.r$ (*Fig. 5*) for different twist angles φ.

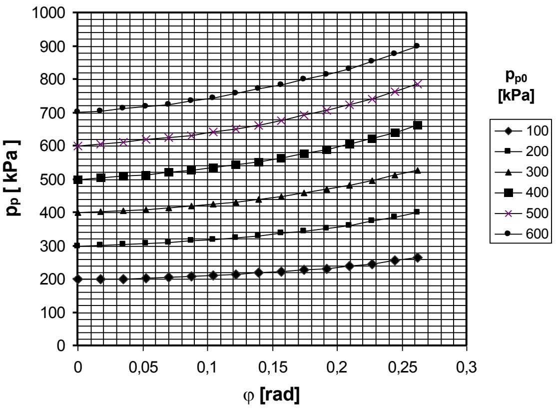

Fig. 3. Overpressure twist angle graph by different initial overpressures
Rys. 3. Zależność podciśnienia od kąta skrętu przy różnych podciśnieniach początkowych

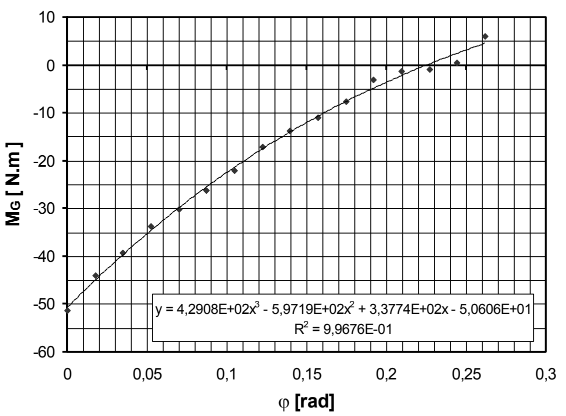

Fig. 4. Torque from rubber shell M_G twist angle graph
Rys. 4. Zależność momentu gumowego płaszcza M_G od kąta skrętu

If we substitute (5) as torque from overpressure of gaseous medium M_V into (1), then we get formula for the torque of coupling:

$$M_S = M_G + p_p \cdot S_e \cdot r \tag{6}$$

If we measure the static load characteristics then the speed of volume change very small. In this case the volume change of gaseous medium can be considered as isothermal [5]. Then applies (7), where V_0 and p_0 are volume of compression space and pressure of gaseous medium by initial twist angle $\varphi = 0$. The dependency of ratio $V/V_0 = p_0/p$ on the static twist angle is determined from the measured values of overpressure (*Fig. 6*).

After derivation, modification and substitution of (4) we get the formula for calculating the volume of compression space by zero twist angle (8).

$$\frac{V}{V_0} = \frac{p_0}{p} \tag{7}$$

$$V_0 = \frac{-S_e \cdot r}{\left[\dfrac{d\left(\dfrac{V}{V_0}\right)}{d\varphi}\right]} \tag{8}$$

Derivation V/V_0 by twist angle φ in the denominator of (8) is determined by derivation of regression polynomial from (*Fig. 6*).

The volume of compression space by initial twist angle calculated according to formula (8) from the ratio of regression polynomials is presented on (*Fig. 7*). It is obvious that volume V_0 should be constant, but in our case it shows a dependence on angle φ, what is probably caused by neglecting of some influences as friction in the bearings of measuring device, flexibility of shafts and hubs, volume change caused by pressure change etc.

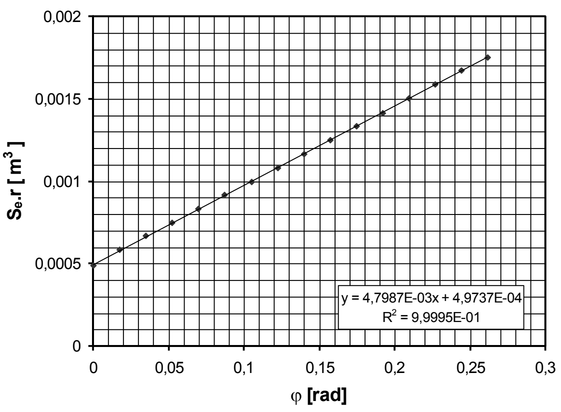

Fig. 5. Static moment $S_e.r$ twist angle graph
Rys. 5. Zależność momentu statycznego $S_e.r$ od kąta skrętu

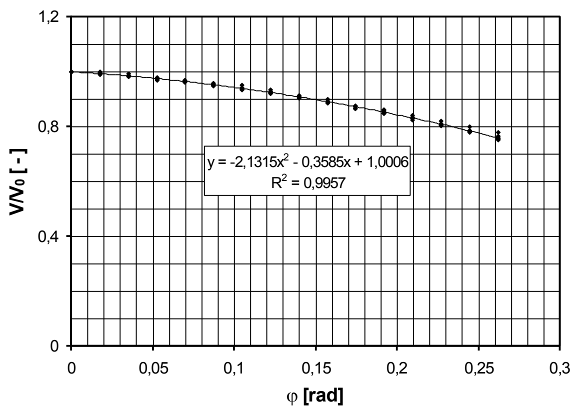

Fig. 6. Ratio V/V_0 twist angle graph
Rys. 6. Zależność stosunku V/V_0 od kąta skrętu

Provisionally V_0 = 1,24869.10^{-3} m^3 is determined as average value from curve on (*Fig. 7*).

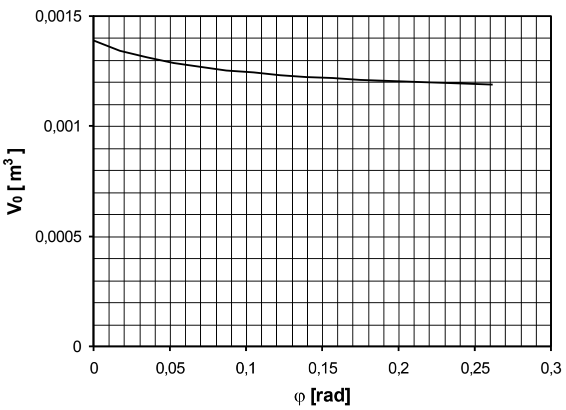

Fig. 7. Volume V_0 twist angle graph
Rys. 7. Zależność objętości V_0 od kąta skrętu

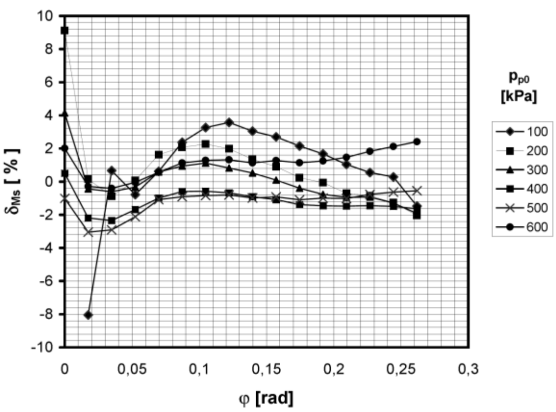

Fig. 8. Percentage difference between measured and computed values
Rys. 8. Procentowa różnica między zarejestrowanymi a obliczonymi wartościami

The volume of compression space is calculated by (9). After substituting (2), (7) and (9) into (5) and modifications we get the formula for computing the torque from overpressure of gaseous medium in the compression space (10). Where p_{p0} is the initial overpressure of gaseous medium by zero twist angle.

$$V = V_0 - \int_0^{\varphi} S_e \cdot r \cdot d\varphi \qquad (9)$$

$$M_V = \left[\left(p_{p0} + p_a\right) \cdot \frac{V_0}{V_0 - \int_0^{\varphi} S_e \cdot r \cdot d\varphi} - p_a \right] \cdot S_e \cdot r \qquad (10)$$

For improving the accuracy of model we can change the provisional value of V_0 by minimizing the cubes of percentage differences between computed values (M_{Scomp}) and measured values (M_{Sexp}) of static load torque δ_{Ms} = 100.(M_{scomp} – M_{sexp})/M_{Sexp}. In this way we obtain the optimal value V_0 = 1,2764.10^{-3} m^3. The values of percentage differences between computed and measured values δM_s for investigated coupling are shown on (*Fig. 8*). The differences $\delta_{Ms} \cong$ (-8% to +9%) we consider in practical terms as acceptable.

It is necessary to mention, that value δ_{Ms} was not computed for overpressure p_{p0} = 100 kPa by twist angle $\varphi = 0$, because at this point is the measured value M_S zero (δ_{Ms} would be infinite).

5. CONCLUSION

By comparing the percentage difference δ_{Ms} (*Fig. 8*), as well as the correlation coefficient (R=0,99972) between computed and measured values we can say that the presented mathematical and physical model for given pneumatic flexible shaft coupling shows a very good agreement with experimentally obtained values of static load torque. This mathematical and physical model can be adapted to different modes of gaseous medium filling (out of operation by zero load, by operation), for couplings with different lengths and diameters of filling and interconnecting tubes (by modifying the volume V_0).

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