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## **A METHOD OF LOADING UNIT FORMATION TAKING INTO ACCOUNT MASS, LOAD-BEARING STRENGTH AND SURFACES OF PACKING UNITS**

**Summary.** The problem of loading unit formation is computationally complex in nature. This article presents a heuristic algorithm of forming unit loads, which can be applied to unit load arrangement on unit load devices. This method accounts for dimensional, mass and load-bearing strength of loading units and loading devices. Moreover, the rotation of packages about a 90° vertical axis has been made possible. In this algorithm, the bearing surface of each packing unit is entirely supported. This guarantees the stability of additional unit load layers. A sample calculation of the arrangement of 30-unit loads is presented in this article.

**Keywords:** forming loading units; heuristic algorithm of loading unit formation; load-bearing strength of packing units

### **1. INTRODUCTION**

The arrangement of loading units is one of the more challenging problems to solve in the field of transport due to its computational complexity. For problems with medium to large

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quantities of loading units, it becomes impossible to obtain solutions that better utilize the use of loading space and, at the same time, guarantee cargo safety. To date, the optimization problem models and heuristic methods, which have been developed, have not been able to correctly obtain solutions for all decision alternatives. By and large, the results obtained have erroneous physical interpretations. Moreover, the existing models do not account for packing unit-bearing surface support, mass and the load-bearing strength of both packing units and unit load devices.

The optimization problem models, which do result in accurate physical interpretations [11, 12, 21], are characterized by long solving times for problems with small quantities of packing units. For this reason, existing models cannot always be applied to logistics systems as support systems.

The commonly used empirical approach, which requires a shorter execution time to obtain unit load formation solutions, offers significant advantages over complex optimization methods or even heuristic methods in filling loading devices. Utilizing this approach, however, results in an increased risk of damaging unit loads due to the excessive weight of stacked packing units.

Taking all of the above into account, studies were undertaken to develop a new heuristic method of forming unit loads. The obtained results are presented and described in this article.

## 2. LITERATURE REVIEW

Heuristic methods, by nature, contain simplifying assumptions. In the case of unit load formation problems, they can be divided into three methods: guaranteeing or failing to guarantee packing unit-bearing surface support, considering or failing to consider the rotation of packing units, and taking into account or failing to take into account the load-bearing strength and durability of packing units.

Heuristic methods, in which rotation of packing units is possible about an axis, have been proposed by Gehring and Bortfeldt [9], Wang, Li and Levy [19], Wu, Li, Goh and de Souza [20], and Engeblad and Pisinger [8].

Engeblad and Pisinger [8] developed an iterative heuristic method for a two-dimensional packing problem based on a method of two permutations containing packing unit numbers described in [14]. Additionally, building on an original algorithm in [8], a heuristic method for a three-dimensional packing problem was proposed, where the packing units arranged in a loading device represent three permutations respective to each dimension. This algorithm did not account for load-bearing strength, durability and mass of packing units. Complete support for bearing surfaces of packing units was also not included in this method.

Another approach to solving this problem was presented by Wu, Li, Goh and de Souza [20]. The authors applied two genetic algorithms in order to solve a three-dimensional packing problem for cuboid packing units. With the help of the first algorithm, it is possible to obtain a solution for a single-unit load formation. The second algorithm allows for many unit loads to be formed from various sized packing units and loading devices. These algorithms can be used to form unit loads by solely considering the size of packing units. The remaining parameters, for example, mass and the load-bearing strength and durability of packing units, are not taken into account. Moreover, the complete support of bearing surfaces is not ensured, which fails to guarantee the stability of the formed loading units.

A solution for arranging pallets in a container was proposed by Sheng, Hongxia, Xisong and Changjian [18]. In an effort to better utilize loading space in a container, where additional unit loads cannot fit, the arrangement of packing units was taken into account.

A heuristic method composed of two algorithms, the binary search tree and the greedy algorithm, was developed. First, the binary search tree arranges unit loads in a container. Next, using the greedy algorithm, packing units, which did not fit, are positioned in the remaining free space. In this heuristic method, it is possible to rotate packing units and loading units about three axes. Moreover, the arranged packing units and unit loads have adequate support for their bearing surfaces.

Aside from packing unit rotation, the stability of the arranged load is also necessary in order to obtain practical solutions of the unit load formation problem. Methods that take this into consideration were proposed by Gehring and Bortfeld [9], Bischoff and Ratcliff [3,4], and Bischoff, Janetz and Ratcliff [2]. An extension of this class of model is presented in [16], where the maximum number of layers that a selected packing unit may withstand was defined. This, however, may solely be applied with respect to homogenous packing units. Arranging additional layers of packages in a loading space requires knowledge of both mass and load-bearing strength of the packing units.

Despite the importance of heuristic methods available in literature, which contain constraints preventing damage to packing units, due to their load-bearing strength, ensuring proper bearing surface support for packing units has been neglected. In these methods, each packing unit is characterized by a centre of mass and load-bearing strength, depending on its placement.

An example of a method that addresses the load-bearing sides of packing units can be found in [2]. It was proposed that packing units should be arranged into a package subset by layer, beginning with the load-bearing surface of the loading device. The following layers should be added until the loading space becomes maximally filled. Each package subset is made up of one type of packing unit, which is arranged length- and widthwise without stacking. This model proposed arranging a maximum of two package subset types in each layer. The authors suggested this in an attempt to tackle the problem of varying heights in each individual layer.

In the heuristic method, accounting for the load-bearing strength of packing units is less complicated if the units have identical mass and strength characteristics. In this case, placing packages directly on top of another does not require calculating the sum of individual masses, dependent on the bearing support surface of packing units. Methods that reflect this concept have been introduced by Liu, Lin and Yu [13], and Gendreau, Iori, Laporte and Martello [10].

In contrast to the method described in [13], Bischoff [1] acknowledged the fact that packing units not only vary in shape and size, but also in load-bearing strength. In this method, complete load-bearing surface support of packing units was taken into consideration. The available loading space is recorded in two matrices. The first matrix contains data pertaining to the height of the available loading space and the second contains the load-bearing strengths.

A similar approach to the method described in [1] was presented by Gehring and Bortfeldt [9]. Five constraints were utilized in the genetic algorithm for forming container unit loads developed by the authors. The first constraint allowed for the ability to rotate packing units. A portion of the packing units can be rotated about three axes; however, the remaining packing units can only be rotated about a horizontal axis. The next two constraints enabled a package to be placed in a column, due to the load-bearing strength of the supporting packages. These columns do not transfer their vertical weight and are formed from heterogeneous packing

units in a way that maximizes the use of container space. The next constraint in the algorithm is responsible for load stability, which is dependent on the support of bearing surfaces and the determination of the centre of mass for container loading units. It is assumed that the centre of mass of packing units is located at their geometric centre. Thus, the packing units are stable, as long as the entire load-bearing surface is directly supported by another packing unit.

The suggested approach in [1] seems to be simplified due to the fact that the possibility to directly support packing unit surfaces with multiple packing units is omitted. Additionally, the assumption that the packing unit's mass is located at the geometric centre and its resulting transfer of force is independent of the supporting surface. As a result of this simplification, the weight of each loading unit is transferred through the loading unit that supports the geometric centre of its base. Therefore, it is justified to claim that this model should be further developed in order to include the vertical weight transfer under the condition that the mass of each packing unit should be distributed throughout the entire surface of its base.

Similarly to the approach described in [1] and [9], Ratcliff and Bischoff [17] presented their method, which included an algorithm presented in [2]. In their approach, they considered the vertical weight transferred through packing units, dependent on the bearing surface support of the remaining packing units. Despite the fact that this method takes into account the load-bearing strength of each packing unit, the vertical load-bearing strength is only identified for individual layers of packages. This ensures the safety of packing units; however, it is obtained at the expense of efficiently utilizing loading space. In order to improve the quality of solutions, it is necessary to make the load-bearing strength of packing units independent of the average strength of cargo layers.

Most of the heuristic methods found in the literature [5, 6, 7, 14] make it possible to obtain a solution that maximally fills a loading space. By and large, in heuristic methods, the ability to transfer weight through packing units is neglected. In the methods where this is considered, the bearing surface support of packing units or the ineffective use of loading space is overlooked more than once.

In this way, the need to formulate a heuristic algorithm for forming unit loads, in which the maximum load-bearing weight of each packing unit is considered, as well as the acceptable level up to which formed unit loads will be filled, is justified.

### 3. APPROACH TO THE PROBLEM

The formulated algorithm is a proprietary method of arranging packing units (PUs) in/on loading devices (LDs). This method is free from limitations described in the literature of heuristic algorithms, which fail to simultaneously consider the possibility of rotating PU, by transferring vertical weight through PUs and LDs, as well as adequately supporting the load-bearing surfaces of PUs. A flowchart of the algorithm developed for a formulated model is presented in Fig. 1.

For the purpose of designing a heuristic algorithm, a formalism was introduced, which includes:

- a set of numbered PUs is  $PO = \{1, \dots, i, \dots, n\}$ , where  $n$  is the quantity of PUs and  $i$  is a PU number
- a set of numbered LDs is  $UL = \{1, \dots, j, \dots, m\}$ , where  $m$  is the quantity of LDs and  $j$  is the number of LDs
- a set of numbered PU sorting methods is  $MS = \{1, \dots, \zeta, \dots, \sigma\}$ , where  $\sigma$  is the quantity of PU sorting methods and  $\zeta$  is the method number

- a set of numbered PUs placed in the  $j$ -th LD for the  $\zeta$ -th PU sorting method in the form  $SUPO(\zeta, j) = \{i: i \in PO, e_{ij}^{\zeta} = 1\}$ ,  $\zeta \in MS, j \in UL$ , where  $e_{ij}^{\zeta} = 1$ , when the  $\zeta$ -th sorting method of the  $i$ -th PU is arranged on the  $j$ -th LD; in the opposite case,  $e_{ij}^{\zeta} = 0$ , where  $i \equiv \eta_{\gamma} \in VPU$
- a set of numbered PUs not arranged on any LD for the  $\zeta$ -th PU sorting method  $SUNPO(\zeta) = \{i: i \in PO, \sum_{j \in UL} e_{ij}^{\zeta} = 0\}$ ,  $\zeta \in MS$
- a vector of numbered PUs is  $VPU_s = [\eta_1, \dots, \eta_{\gamma}, \dots, \eta_{\delta}]$ , where  $\delta$  is the quantity of PUs and vector components  $VPU_s$ ,  $\gamma$  is the numbered vector component  $VPU$ , and  $\eta_{\gamma}$  is a numbered PU such that  $i \equiv \eta_{\gamma}, i \in PO$ ,
- a vector of numbered LDs is  $VLD_s = [\lambda_1, \dots, \lambda_j, \dots, \lambda_m]$ , where  $m$  is the number of LDs and vector components  $VLD_s$ , and  $j$  is the numbered vector component  $VLD$ , such that  $j \equiv \lambda_j, j \in UL$
- the binary variable  $\tau_{\zeta i} = 1$ , when, at the  $\zeta$ -th sorting method, the  $i$ -th PU is rotated  $90^\circ$  about a vertical axis; in the opposite case,  $\tau_{\zeta i} = 0$ ,  $i \in PO, \zeta \in MS$

In the first step of the algorithm (Fig. 1) the decisive situation of forming loading units is determined and the PU characteristics are introduced. The types of significant constraints can be selected here, including whether or not the transfer of vertical weight is to be considered. The required PU characteristics are adequate for the selected decision. Only a selected variant of the algorithm is shown in Fig. 1.

In the second step, the method of forming unit loads is selected, depending on the result of the decision. In the algorithm, there are two different models of arranging PUs in/on LDs, which are selected depending on the decision that is made. The first model takes into account the load-bearing strength and mass (Fig. 1), while these parameters are omitted in the second.

Next, according to the selected criterion, the  $PO$  set is sorted. There are six methods of sorting PUs in the  $MS$  set, which includes increasing and decreasing sorting according to mass, size and load-bearing surface characteristics.

Following this, individual elements of the  $PO$  set are assigned to a vector component  $VPU$  in such a way that  $\eta_{\gamma} = i, i \in PO$ . Then, starting with the first ( $\eta_{\gamma=1}$ ) vector component  $VPU$ , the possibility of placing its corresponding PU on the  $\lambda_{j=1}$ -th LD ( $VLD$  vector component) is checked. When placing a PU on the  $\lambda_j$ -th LD, it is added to the  $SUPO(\zeta, j)$  set. If the LD lacks space regarding the desired characteristics, a potential placement in the next  $\lambda_{j+1}$  LD is checked. However, if the PU cannot be placed into any of the LDs, it is added to the  $SUNPO(\zeta)$  set. The possibility of placing the remaining PU in the  $\zeta$ -th sorting method is examined. Each following  $\zeta$ -th sorting methods of PU arrangement on/in LDs is repeated and the  $VPU$  vector is redefined. When the sorting method  $\zeta \geq \sigma$ , the algorithm stops arranging PUs and selects the best solution for formed loading units based on the given evaluation criteria.

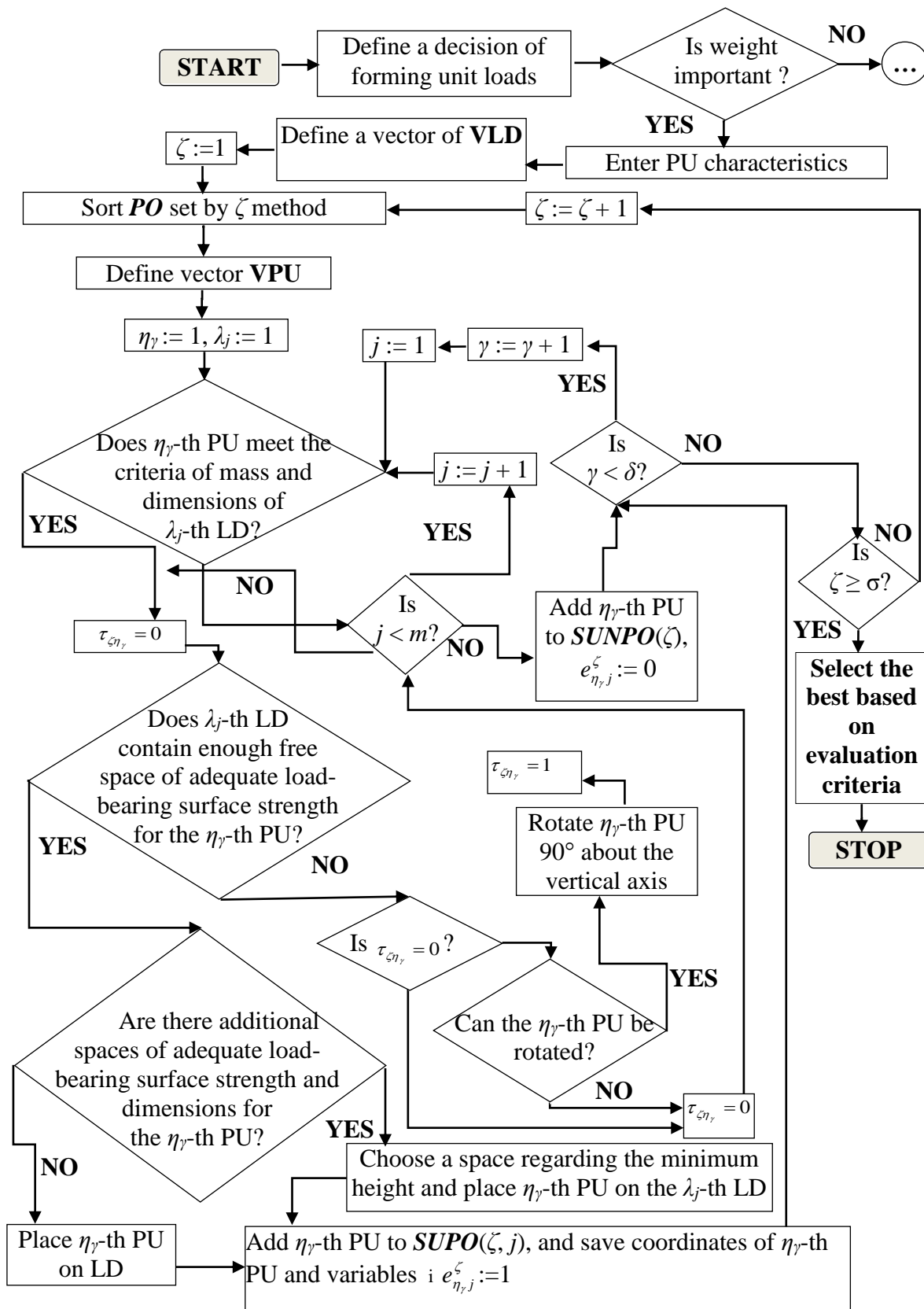


Fig. 1. Outline of a heuristic algorithm for forming loading units while taking into account load-bearing strength and mass

#### 4. COMPUTATIONAL RESULTS

The heuristic method for forming unit loads described in this article was implemented in the Python programming language. Next, many sample calculations were defined and solved in order to assess the method's effectiveness and to verify accuracy. The obtained results were referred to analytical solutions of the problem.

This article describes one of these examples. A problem of arranging 30 packing units on Euro-pallets (1,200 mm x 800 mm x 144 mm in size) was solved. The maximum height of each pallet unit load was set to 1,344 mm, while the maximum load could not exceed 1,000 kg. The parameters of packing units arranged on the pallet are listed in Table 1.

Tab. 1

Packing unit characteristics

PU number (i)	Length of PU [mm]	Width of PU [mm]	Height of PU [mm]	Mass of PU [kg]	Maximum load-bearing strength of PU [kg]
1	308	193	191	9	80
2	142	366	272	11	74
3	266	172	364	27	82
4	420	185	487	19	46
5	221	505	158	2	84
6	386	156	224	17	31
7	537	588	350	27	53
8	505	194	172	27	20
9	397	305	218	8	27
10	500	300	292	26	81
11	545	503	327	12	97
12	536	134	480	10	28
13	526	202	140	15	90
14	286	312	171	24	66
15	355	304	343	4	54
16	492	212	339	29	15
17	258	162	233	9	10
18	413	126	318	4	15
19	394	583	279	6	10
20	145	451	175	26	30
21	348	388	230	6	10
22	377	104	103	24	51
23	369	333	315	25	50
24	300	329	300	18	54
25	100	100	486	5	80
26	393	122	234	18	78
27	184	201	165	9	80
28	382	128	332	16	52
29	161	408	104	23	51
30	230	448	217	13	30

As a result of solving a sample problem using the heuristic method, an arrangement of packing units on two pallets was obtained. Packing units numbered 1-9 and 11-15 were placed on the first pallet, as illustrated in Fig. 2. On the second pallet, shown in Fig. 3, packing units numbered 10 and 16-30 were arranged. The solving time for this problem was 2 s. In the solution, the conditions of bearing surface support while stacking and the possibility of rotation were also met.

On the basis of the obtained results, the proposed heuristic algorithm of forming loading units was found to be valid. In addition to yielding a realistic solution (a logical physical interpretation), it is possible to create an accurate visual representation of pallet loading units.

However, after analysing the solutions to various problems, it has been concluded that additional improvements should be made to the packaging unit sorting algorithm, as well as other elements of the algorithm in order to better utilize the loading space.

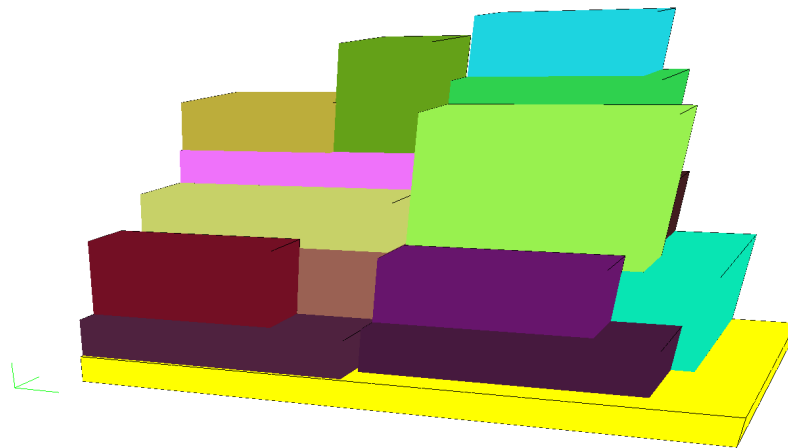


Fig. 2. The first pallet loading unit

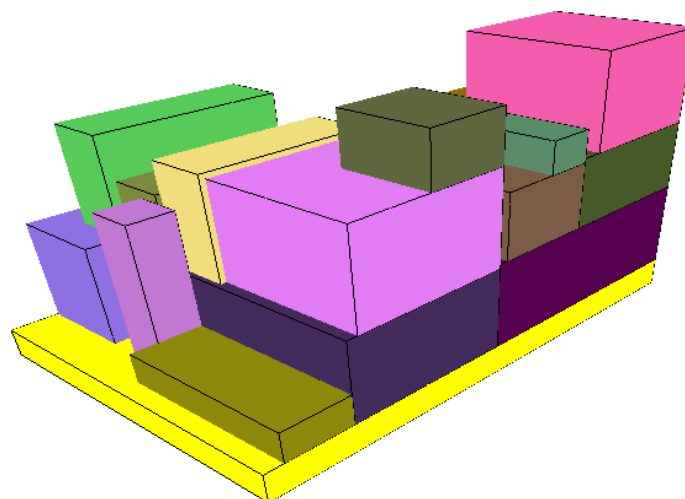


Fig. 3. The first pallet loading unit



## 5. CONCLUSION

With the help of a formulated implementation of the heuristic algorithm, it is possible to form unit loads in which the packaging units may be heterogeneous with regard to size, mass and load-bearing strength. In this method, the support of load-bearing surfaces and vertical weight transfer, dependent on bearing surfaces, was ensured. Additionally, the limitations of loading device load-bearing strength and vertical weight transfer, as well as the possibility to rotate packing units by 90° about a vertical axis, were taken into account.

This tool can effectively form loading units for sample problems of up to 1,000 packing units. Moreover, with the help of the computer implementation of this heuristic algorithm, it is possible to solve sample problems for selected decisions, whose solutions may be applied in the logistics industry.

Due to the advantages of using a heuristic method to obtain solutions for a large amount of packing units in a short resolution time, additional changes can be made to the packing unit sorting method, while the possibility of supporting the load-bearing surfaces of packing units with multiple packing units can be introduced to further improve results. These changes should considerably improve the degree of loading space utilization and make it applicable to solving a wide range of complex packing unit arrangement problems.

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